





MULTIPLICATION AND DIVISION SEQUENTIAL LEARNING OSMOTHERLEY CP

Revised September 2021

Highlighted= 2020 Maths guidance ready-to-progress criteria

Year group	sequence	methods
FOUNDATION	*to explore and represent patterns within numbers up to ten, including evens and odds, double facts and how quantities can be distributed equally	*use visual pictures and practical apparatus to double, then halve and share and to explore odds and evens (numicon)
ONE	<p>*solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</p> <p>*count in multiples of 2,5,10</p>	<p>Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.</p> <p>They make connections between arrays, number patterns, and counting in twos, fives and tens.</p>  <p>2 groups of 3 are 6 ($3 + 3$)</p> <p>3 groups of 2 are 6 ($2 + 2 + 2$)</p> <p>Children should experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. role play, counters, cubes etc.</p> <p>Children will explore everyday versions of arrays such as egg boxes, baking trays, ice cube trays and wrapping paper</p>

		<p>Children will use repeated addition to carry out multiplication supported by the use of counters/cubes/coins/number lines</p> <p>$10p + 10p + 10p + 10p + 10p = 30p$</p> <p> 5 hops of 10</p> <p> $5 \times 10p = 50p$</p> <p>Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication.</p> <p></p>
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TWO	<p>*recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2 5 and 10 multiplication tables</p> <p>* recognise repeated recall and use multiplication and division facts for the 2, 5</p>	<p>Pupils use a variety of language to describe multiplication and division.</p> <p>Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use</p>
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and 10 multiplication tables, including recognising odd and even numbers and also counting in multiples of 3

* calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs

* show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

* solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts


***relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor and to division equations (quotative division)**

other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

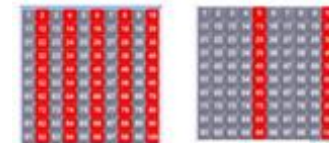
Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, $4 \times 5 = 20$ and $20 \div 5 = 4$).

Repeated Addition

'Count out three groups of 5 then count the beads altogether'.

$$5 + 5 + 5 = 15$$



Children explore patterns on a 100 square to help them begin to recognise multiples and rules of divisibility.

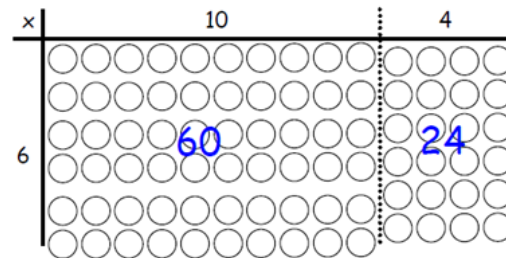


Multiples of 2

Multiples of 5

Numicon can be used on a number line to develop understanding of multiplication as repeated addition and can be made into arrays.

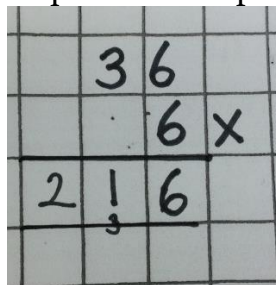
THREE	<p>*apply known multiplication and division facts to solve contextual problems with different structures including quotative and partitive problems</p> <p>*recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables and count in multiples of 4,8,50,100</p> <p>.</p> <p>*write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including two-digit numbers times one-digit numbers, using mental methods and progressing to formal written methods</p> <p>*pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division</p> <p>* solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects</p>	<p>Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency.</p> <p><u>Multiplication Sequence Of Methods</u></p> <p>A. Pupils multiply a 2 digit number by a single digit and practically partition the two digit numbers - extending use of arrays ,arrow cards, base ten to support this method with practical apparatus</p> <p>e.g 13×4</p>  <p>e.g 14×6</p>



B.

Moving to formal methods for short multiplication the children “think digit name” reinforced by base ten e.g 4×3 “ten rods” = 12 “ten rods” and carry on the bottom line of answer box to distinguish from addition (although optional for children who need visual top-line-carried-numbers to reinforce the step of adding carried number)

Sequence: multiplier x ones digit then multiplier x tens digit



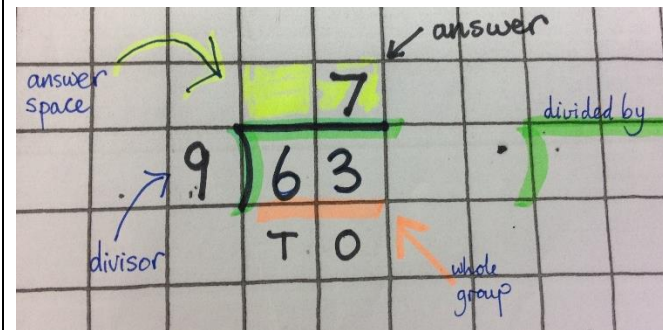
Division Sequence Of Methods

A. Using x tables knowledge e.g

$144 \div 12$ so “How many twelves go into 144?”

B. Children write and solve as linear calculation then transfer answer to “bus stop” short division layout to understand positioning

e.g $63 \div 9 = 7$ and then transfer to layout:



C. Pupils use short division method (directly using “bus stop” layout)

e.g $98 \div 7 = 14$

Children understand and can demonstrate using practical apparatus- “9 base ten rods divided into groups of 7? Can only make **one** group so remainder rods(2) need to be exchanged into ones and carried into ones column. Now 28 ones divided into groups of 7: each group has **four**” answer **14**

Children move on to using their x tables recall to use formal method - “How many sevens into 9? **One** remainder **2** to carry. How many sevens into 28? **four**” answer 14
 $98 \div 7$ becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

Answer: 14

Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (for example, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

FOUR

***multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size**

*recall multiplication and division facts for multiplication tables up to 12×12 (and count in multiples of 6,7,9,25,1000)

*use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

*recognise and use factor pairs and commutativity in mental calculations

*multiply two-digit and three-digit numbers by a one-digit number using formal written layout (**short multiplication**)

***solve problems involving multiplying and adding, including and understanding the use of the distributive law** to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

***manipulate multiplication and division equations, and understand and apply the commutative property of multiplication**

Before carrying out calculations children should be encouraged to estimate their answer using rounding.

They should compare their answer with the estimate to check for reasonableness.

Short Multiplication (single digit multiplier)

"Think digit name"

	2	4	3	
			4	x
<hr/>				
	9	7	2	
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Distributive Law

E.g

3 "lots" of **(2+5)** is the same as 3 "lots of" 2

plus 3 lots of 5

So, the **3**× can be "distributed" across the **2+5**, into **3**×**2** and **3**×**5**

6 × 404 ?

$$\begin{aligned}6 \times 404 &= 6 \times 400 + 6 \times 4 \\ &= 2400 + 24 \\ &= 2424\end{aligned}$$

$$16 \times 6 + 16 \times 4?$$

$$\begin{aligned} 16 \times 6 + 16 \times 4 &= 16 \times (6+4) \\ &= 16 \times 10 \\ &= 160 \end{aligned}$$

Short Division

98 ÷ 7 becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 86 \text{ r} 2 \\ 5 \overline{) 432} \end{array}$$

Answer: 86 remainder 2

FIVE

***multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size.**

***identify multiples and factors, including finding common multiples, all factor pairs of a number, and common factors of two numbers and express a given number as a product of 2 or 3 factors**

*know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

*establish whether a number up to 100 is prime and recall prime numbers up to 19

***multiply numbers using formal written methods up to 4 digits by a one digit number (short multiplication) or 4 digits by a two-digit number (long multiplication) using**

*multiply and divide numbers mentally drawing upon known facts

***divide numbers up to 4 digits by a one-digit number** using the formal written method of short division and interpret remainders appropriately for the context

*multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

*recognise and use square numbers and cube numbers, and notation for squared (2) and cubed (3)

*solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

Pupils practise and extend their use of the formal written methods to multiply and divide:

Short multiplication (using single digit divisor) "Think digit name"

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \end{array}$$

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \end{array}$$

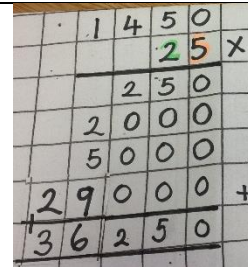
Long multiplication (using more than one digit divisor)
Pupils are mindful of digit values and the partitioning of multipliers

- A. Use jottings alongside calculations at first. Look for mental maths shortcuts e.g whole four digit number multiplied by ten

$$\begin{array}{r} 2346 \\ \times 14 \\ \hline 9384 \quad (4 \times 6) \\ 93840 \quad (4 \times 40) \\ 234600 \quad (4 \times 300) \\ 2346000 \quad (4 \times 2000) \\ \hline 32844 \quad (10 \times 2346) + \end{array}$$

- B. Moving on to method of omitting jottings and finding more mental maths shortcuts

*solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
 *solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates



Division

Remainders can be expressed as whole number or fraction

432 ÷ 5 becomes

$$\begin{array}{r} 86 \text{ r} 2 \\ 5 \overline{) 432} \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 45 \text{ r} 1 \\ 11 \overline{) 496} \end{array}$$

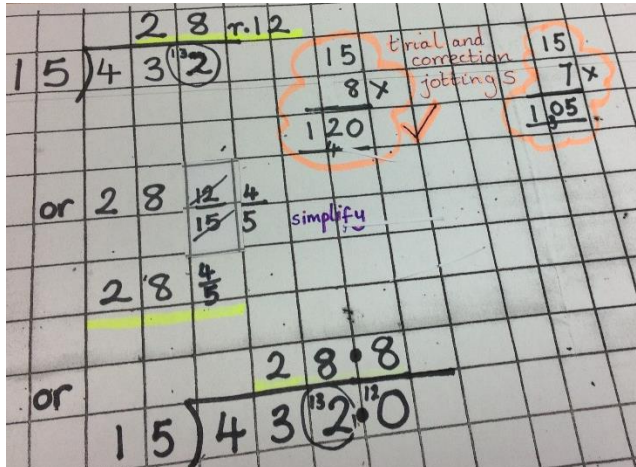
Answer: $45\frac{1}{11}$

Pupils apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by **expressing results in different ways according to the context**, including with remainders, as fractions, as decimals or by rounding

(for example, $98 \div 4 = 24 \text{ r} 2 = 24\frac{2}{4} = 24.5 \approx 25$).

		<p>Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres. Distributivity can be expressed as $a(b + c) = ab + ac$. They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, $4 \times 35 = 2 \times 2 \times 35$; $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$).</p> <p>Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, $13 + 24 = 12 + 25$ $35 = 5y$)</p>
<p>SIX</p>	<p>multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</p> <p>*divide numbers up to 4 digits by a two-digit whole number using the formal written method of short division or long division where appropriate, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p> <p>*perform mental calculations, including with mixed operations and large numbers</p> <p>*identify common factors, common multiples and prime numbers</p> <p>*use their knowledge of the order of operations to carry out calculations involving the four operations</p>	<p><u>Long Division Using Trial And Correction Inverse Multiplication Jottings (expressing remainders in different ways)</u></p>  <p>Handwritten work on grid paper illustrating three methods for dividing 28.8 by 15:</p> <ul style="list-style-type: none"> Method 1: Long division showing $15 \overline{) 432}$ with a remainder of 12. Method 2: Trial and correction using multiplication: $15 \times 8 = 120$ and $15 \times 7 = 105$. A checkmark is placed under the correct result. Method 3: Simplification of the fraction $\frac{28.8}{15} = \frac{4.8}{2.5}$.

“How many fifteens into 4? Zero. How many fifteens into 43? Two. Carry the remainder of 13. How many fifteens into 132? Use trial and correction jottings to find the answer. 8 remainder 12. What is the final answer? Twenty eight remainder twelve.

Note: remainders expressed as whole number/fraction/decimal

Where division takes place in real life context:

e.g: coaches hold 72 passengers and we need to take 78 pupils and 2 teachers, 2 T.A's and a volunteer helper on a trip. How many coaches do we need?

Two coaches are needed even if only a handful of seats are filled on the 2nd coach because **everyone** has to be transported so pupils round up as would be necessary in real life context.